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Probabilistic Modeling of Passengers and Carriers Preferences via Bicriterial Approach ⋆

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Abstract: The problem of modeling preferences of passengers and carriers in conditions of changing the transport network structure is considered. The patterns of passengers and carriers preferences of are formulated on the basis of the bicriterial problem solution. The choice of a route by a random passenger (carrier) is considered as an optimization problem with a random objective function. Forecasting the distribution of traffic flows after changing the structure of the network is carried out within the framework of the Markov chain model. The correspondence matrix of the original network and patterns of passenger preferences are used to estimate the transition probabilities.

Keywords: Transport network, network structure change, bicriterial problem, transition probabilities.

1. INTRODUCTION

We consider a mathematical model for choosing a mode of transportation for a passenger when the structure of a transport network is changed. Forecasting of passenger traffic is closely related to the analysis of the correspondence matrix, which elements reflect the intensity of traffic and depend on various factors, see Haight (1963). In the well-known article de Rus, Inglada (1997) a mathematical model was proposed to describe the change in the generalized cost index when high-speed railroads were put into operation. The article de Rus (2012) provides a detailed analysis of the change in the generalized cost of the trip (taking into account the travel time and ticket price) when high-speed railways are introduced in Spain.

2. FORMALIZATION OF PASSENGER PREFERENCE PROBLEM

Let on an existing arc of the transport network (for example, connecting two relatively large cities) is possible to use several modes of transport.

The passenger has a choice between possible alternatives: $e_1, \dots, e_4$, where e_1 is aviation, e_2 is automobile, e_3 is interurban bus transport, e_4 is ordinary railway.

We will consider the expansion of the transport network by adding high-speed links, that is, $A_1 = A_0 \cup e_5$, where e_5 is the new link high–speed rail transport. We will describe the preferences of a randomly selected passenger with a random vector $\{X_0, X_1\}$, where X_i are discrete random variables that depend on each other and take the values $X_i \in A_i$, $i = 1, 2$. Here X_0 is the distribution of preferences before the introduction of a new link in the transport network and X_1 after the introduction.

The statistics show Martynenko, Petrov (2018) that the distribution of the random vector $\{X_0, X_1\}$ depends on the value of the travel time criteria $T(X_i)$ and trip costs $C(X_i)$. In addition, X_1 depends on X_0.

Through vector $q^{(0)} = \{q_1^{(0)}, \dots, q_4^{(0)}\}$ we denote the initial probability distribution of $q_i^{(0)} = Pr\{X_0 = e_i\}$.

Analysis of statistical data shows de Rus, Inglada (1997); Boque (2012), that even in case of double preference, i.e. when one option (for example, air communication) is preferable for both criteria, nevertheless (with reasonable travel times) a part (sometimes small) of passengers prefers other modes of transport.

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3. ESTIMATION OF TRANSITION PROBABILITIES

It is assumed further that Assumption 1 holds.

Assumption 1. A passenger who prefers some kind of transport before changing the structure of the network either retains his choice or changes it to the newly introduced one, if the latter is preferable in terms of the criteria.

Let's denote transition probabilities by

$$p_{ij} = Pr\{X_1 = e_i | X_0 = e_j\}.$$

In view of Assumption 1, the matrix of transition probabilities has the form:

$$P = \{p_{ij}\} = \begin{pmatrix} 1-p_{15} & 0 & 0 & 0 & p_{15} \\ 0 & 1-p_{25} & 0 & 0 & p_{25} \\ 0 & 0 & 1-p_{35} & 0 & p_{35} \\ 0 & 0 & 0 & 1-p_{45} & p_{45} \end{pmatrix}. \quad (1)$$

Thus, we obtain the following equation

$$q^{(1)} = P^T q^{(0)}. \quad (2)$$

Analysis of data on passenger preferences distribution shows that the transition probabilities have a much smaller variation than the initial preferences. An example of estimating the transition probabilities based on data on the preferences of passengers before and after the introduction of the HSR between cities in Europe with a distance of about 400 km is given in the article Timofeeva, Martynenko (2018).

4. BICRITERIAL APPROACH

Let's denote the cost of travel from A to B by each type of transport by c_i, $i = 1, \dots, 4$ and the time spent on moving by each mode of transport by t_i , $i = 1, \dots, 4$.

At this time, not only the travel time is included, but also the time that the passenger spends to, for example, get to and from the airport. If other conditions of travel (such as the convenience of timetables, etc.) are not taken into account, then we get to the optimization problem with two criteria for describing the preferences of one passenger.

$$\begin{array}{l} x_1 + \dots + x_4 = 1, \quad x_i \in \{0; 1\}, \\ T(X) = t_1 x_1 + \dots + t_4 x_4 \to \min, \\ C(X) = c_1 x_1 + \dots + c_4 x_4 \to \min. \end{array} \quad (3)$$

Definition 1. The solution $X^{(1)}$ is preferable to the solution (dominates over the solution) $X^{(2)}$, if one of the 2 conditions holds:

$$T(X^{(1)}) \leq T(X^{(2)}) \wedge C(X^{(1)}) < C(X^{(2)})$$

or

$$T(X^{(1)}) < T(X^{(2)}) \wedge C(X^{(1)}) \leq C(X^{(2)}).$$

The set of non-dominant (Pareto-optimal) solutions of the two-criteria problem (3) is denoted by $E_0 \subseteq A_0$.

For a mathematical description of the probabilistic character of preferences of passengers we will consider the following model.

We assume that passengers choose solutions only from the set E_0. The choice between effective solutions is based on the preference function, which depends on the preferences of a randomly chosen passenger. To describe preferences we will use the "generalized trip cost" introduced in the paper de Rus, Inglada (1997).

In our consideration the generalized trip cost is the sum of two criteria:

$$f(X) = C(X) + \varphi(T(X)),$$

where $\varphi(T) \geq 0$ is "a price" of the time spent by the passenger. Here $\varphi(T)$ is a non-decreasing function $[0, +\infty) \mapsto [0, +\infty)$. It is proposed further that

$$\varphi(T(X)) = \theta T(X), \quad \theta > 0.$$

Thus we get the optimisation problem with one criterion

$$\begin{array}{l} x_1 + \dots + x_4 = 1, \; x_i \in \{0; 1\}, \\ f(X) = C(X) + \theta T(X) \to \min. \end{array} \quad (4)$$

5. RANDOM PREFERENCE

If the time value of θ for all passengers was the same, then they would choose the same solution, i.e. one mode of transport. However, this does not happen, so we will assume that the value of time costs for different passengers is different, so they choose different solutions. The choice of a random passenger is modeled as an optimization problem with a random objective function (4), whose solution is also random. Let's denote the solution of the problem by $X_0(\theta)$.

The random value $X_0(\theta)$ has a discrete distribution on the set E_0, defined by a distribution of the parameter θ and values of c_i and t_i.

Let on the existing arc of the transport network it become possible to use an additional type of transport (for example, high-speed transport), so a set of alternatives expands and become equal to A_1.

The choice problem remains the same, only the dimensions of the vectors have changed: $C_1 \in R^5$, $T_1 \in R^5$, $e_i \in R^5$, $i = 1, \dots, 5$. If the preference function $f(X)$ remains the same, we obtain the problem:

$$\begin{array}{l} x_1 + \dots + x_5 = 1, \; x_i \in \{0; 1\}, \\ f_1(X) = C_1(X) + \theta T_1(X) \to \min. \end{array} \quad (5)$$

The solution of the problem is denoted by $X_1(\theta)$. The random values $X_0(\theta)$ and $X_1(\theta)$ are related to each other. Let's calculate probabilities $q_j^{(0)}$ $(j = \overline{1,4})$ and $q_j^{(1)}$ $(j = \overline{1,5})$.

Taking into account

$$f(e_j) = c_j + \theta t_j\,, \quad j = \overline{1,5},$$

we get that $X_0 = e_k$ is equivalent to the condition

$$f(e_k) \leq f(e_j)\,, \quad j = \overline{1,4}.$$

The last relation is equivalent to the fulfillment of two inequalities

$$\theta \geq \frac{c_k - c_j}{t_j - t_k} \quad \text{for} \quad t_j > t_k\,,$$

$$\theta \leq \frac{c_j - c_k}{t_k - t_j} \quad \text{for} \quad t_j < t_k\,,$$

for all $j = \overline{1,4}$.

Thus, $X_0 = e_k$ if and only if

$$\Theta_0^L(k) \leq \theta \leq \Theta_0^R(k), \quad (6)$$

where

$$\Theta_0^L(k) = \max_{\substack{j=\overline{1,4} \\ t_j > t_k}} \left\{ \frac{|c_k - c_j|}{|t_j - t_k|} \right\},$$

$$\Theta_0^R(k) = \max_{\substack{j=\overline{1,4} \\ t_j < t_k}} \left\{ \frac{|c_k - c_j|}{|t_j - t_k|} \right\}.$$

Consequently, the probability $q_k^{(0)}$ depends on the distribution of the random variable θ and is determined by the relation

$$q_k^{(0)} = Pr\big\{\Theta_0^L(k) \le \theta \le \Theta_0^R(k)\big\}.$$

Reasoning similarly, we obtain that $X_1 = e_m$ if and only if then when

$$\Theta_1^L(m) \le \theta \le \Theta_1^R(m), \tag{7}$$

where

$$\Theta_1^L(m) = \max_{\substack{j=\overline{1,5} \\ t_j > t_m}} \left\{ \frac{|c_m - c_j|}{|t_j - t_m|} \right\},$$

$$\Theta_1^R(m) = \max_{\substack{j=\overline{1,5} \\ t_j < t_m}} \left\{ \frac{|c_m - c_j|}{|t_j - t_m|} \right\}.$$

Consequently

$$q_m^{(1)} = Pr\big\{\Theta_1^L(m) \le \theta \le \Theta_1^R(m)\big\}.$$

Let us find the transition probabilities p_{km} ($k = \overline{1,4}, m = \overline{1,5}$).

The following assertion is proved.

Proposition 1. If the distribution of the random parameter θ is the same at both stages (ie before and after the network structure change), then $p_{km} = 0$ for $k, m = \overline{1,4}$, $k \ne m$, i.e. Assumption 1 is satisfied and the matrix of transition probabilities has the form (1).

To find the matrix P, it suffices to find p_{k5}. From (6) and (7) we get

$$Pr\big(X_1 = e_5 \wedge X_0 = e_k\big) =$$
$$Pr\big(\max\{\Theta_0^L(k), \Theta_1^L(5)\} \le \theta \le \min\{\Theta_0^R(k), \Theta_1^R(5)\}\big),$$

$$p_{k5} = \frac{Pr\big(X_1 = e_5 \wedge X_0 = e_k\big)}{q_k^{(0)}}.$$

6. THE CASE OF RANDOM TIME AND UNCERTAINTY IN THE COST OF A TRIP

Within the model, passengers choose only Pareto–optimal solutions to the task (3), but in reality this is not quite so. Some (relatively small) part of the passengers prefers the dominant solutions. This can be explained by several reasons.

First, both criteria are, in general, probabilistic or not completely deterministic, i.e. the value of the travel time and especially the fare are not unambiguously determined. It can be assumed that the deviation of the travel time from the mean value is of a probabilistic nature. But for the price of travel, as a rule, there is a system of tariffs (discounts, etc.).

Thus, the fare price is not unambiguously determined and it can be modeled either as an indeterminate nonrandom value taking values from a certain interval or as a random variable having some distribution (perhaps discrete), see Zavalishchin, Timofeeva (2017). Accounting for the uncertainty in the price of the trip (and travel time) leads to a complication of the criteria in the problem of choosing the type of transport (3).

Thus, the statement of the problem (3) is modified to the following

$$\begin{aligned} &x_1 + \ldots + x_4 = 1, \quad x_i \in \{0;1\}, \\ &f_T(X, \omega_1) = T(\omega_1)^\top X \to \min, \\ &f_C(X, \omega_2) = C(\omega_2)^\top X \to \min. \end{aligned} \tag{8}$$

Two–criterion problem has become two–criteria problem with random objective functions.

On the other hand, the preferences of passengers may be due to the presence of additional criteria when choosing a solution (such as convenience, safety, etc.) that are not taken into account in the model. In order to take into account these features, it is possible to introduce an additional criterion (criteria) into the statement of the problem.

7. CONCLUSION

The problem of modeling the preferences of passengers is considered when changing the structure of the transport network by the example of the introduction of a high-speed link. The proposed approach can be extended to a wide class of problems of choosing the optimal route and predicting the correspondence matrix.

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